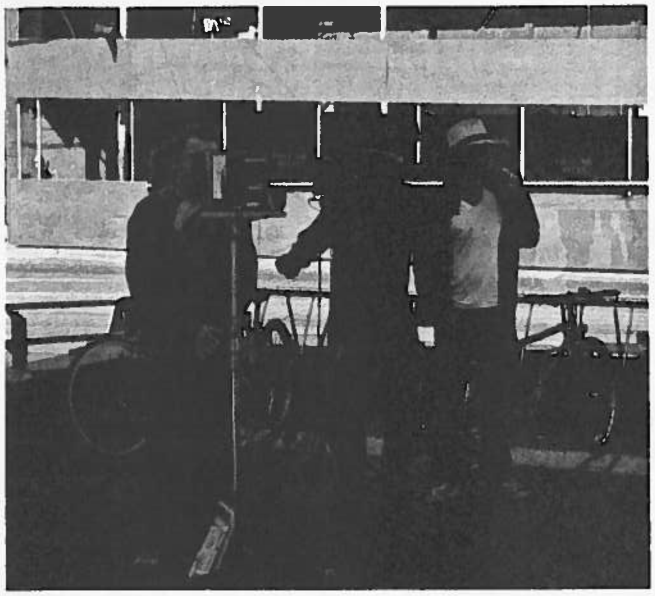
2.4 Equation for a Circle



A licence is not required for portable two-way radios with a power of up to 2 W operating on the General Mobile Radio Service (GMRS) frequencies in Canada. GMRS radios are similar to Family Radio Service (FRS) radios, which are limited to 0.5 W and use different frequencies. Some radios are hybrids that can operate on botl the GMRS and the FRS frequencies.

Investigate

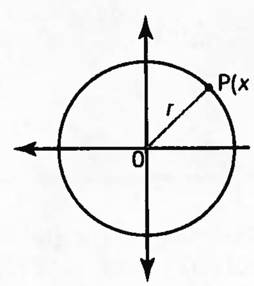
Tools
- grid paper
- compasses

How can you find an equation for a circle?

Near his home, Trevor's GMRS radios have a range of about 5 km.

Method 1: Use Pencil and Paper

1. Draw a circle to represent the range of Trevor's radios. Let the origin represent Trevor's position.
2. Label the x- and y-intercepts of your circle. What do these intercepts have in common?
3. Find four other points on the circle that have integer coordinates Label these points A, B, C, and D, and mark their coordinates on your drawing. Use the distance formula to verify that each of the points is exactly 5 units from the origin.
4. Mark a point P(x, y) anywhere on the circle. Construct a right triangle with OP as the hypotenuse and the rise and the run of OP as the other two sides.



5. Write an equation relating the length of OP to the length of the other two sides of the right triangle. Substitute OP = 5 into the equation.

6. Verify that the coordinates of points A, B, C, and D satisfy the equation in step 5.

7. **Reflect** Will the coordinates of every other point on the circle also satisfy the equation? Explain your reasoning.

8. Away from built-up areas, Trevor finds that his GMRS radios have a range of about 7 km. Add a circle to your drawing to represent this larger range.

9. Write an equation for the larger circle.

10. **Reflect** Write an equation for the circle with centre (0, 0) and radius r. Then, use this equation to write an expression for the radius.

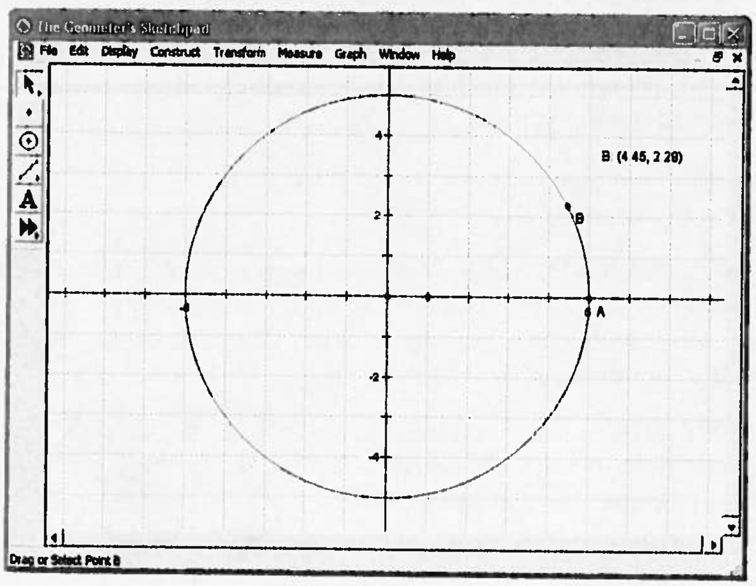
Method 2: Use *The Geometer's Sketchpad®*

1. Choose **Show Grid** from the **Graph** menu.

2. Construct a circle to represent the range of Trevor's radios. Let the origin represent Trevor's position. Use the **Compass Tool** to construct a circle with its centre at the origin and a radius of 5 units.

3. Label the x- and y-intercepts of the circle. What do these points have in common?

4. Use the **Point Tool** to construct a point on the circle. Select the point and choose **Coordinates** from the **Measure** menu. Then, drag the point around the circumference of the circle to find four other points that have integer coordinates. Construct these points and label them with their coordinates.



5. Verify that each of the points is 5 units from the origin. Select a point and the origin, and choose **Distance** from the **Measure** menu. Use the same method to measure the distance from the origin to each of the other points.

Tools
- computer with *The Geometer's Sketchpad®*

6. Construct a point anywhere on the circle. Label the point P. Construct a right triangle with OP as the hypotenuse and the rise and the run of OP as the other two sides.

7. Write an equation relating the length of OP to the length of the other two sides of the right triangle. Substitute OP = 5 into the equation.

8. Verify that the coordinates of the points in step 4 satisfy the equation in step 7.

9. **Reflect** Will the coordinates of every other point on the circle als satisfy the equation? Explain your reasoning.

10. Away from built-up areas, Trevor finds that his GMRS radios hav a range of about 7 km. Add a circle to your drawing to represent this larger range.

11. Write an equation for the larger circle.

12. **Reflect** Write an equation for the circle with centre (0, 0) and radius r. Then, use this equation to write an expression for the radius.

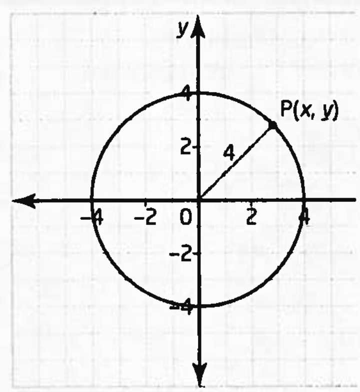
Example 1 Equation for a Circle

Find an equation for the circle with centre (0, 0) and radius 4.

Solution

The distance from the origin to any point P(x, y) on the circle is the length of the radius. So,

OP = 4



The distance formula also gives an expression for the length of OP:

$$\begin{aligned} OP &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(x - 0)^2 + (y - 0)^2} \\ &= \sqrt{x^2 + y^2} \end{aligned}$$

Therefore,

$$\begin{aligned} \sqrt{x^2 + y^2} &= 4 \\ x^2 + y^2 &= 16 \end{aligned}$$

An equation for the circle is $x^2 + y^2 = 16$.

Example 2 Determine Whether a Point Lies Within a Circle

a) Determine an equation and the radius for the circle that has its centre at the origin and passes through the point A(6, −8).

b) Is the point B(−5, 9) inside this circle?

Solution

a) An equation for a circle centred at the origin has the form $x^2 + y^2 = r^2$.

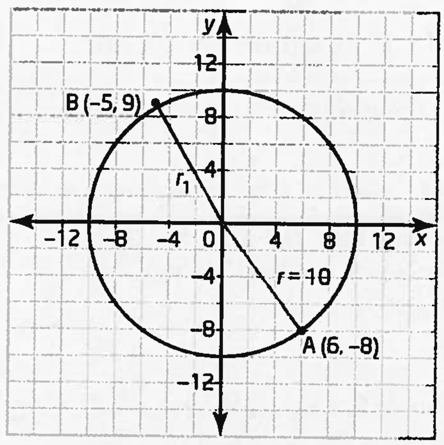
Substitute the coordinates of the point (6, −8) into the equation for the circle.

The point (6, −8) lies on this circle, so the coordinates of the point must satisfy the equation of the circle.

$$\begin{aligned} x^2 + y^2 &= r^2 \\ 6^2 + (-8)^2 &= r^2 \\ 36 + 64 &= r^2 \\ 100 &= r^2 \\ \sqrt{100} &= \sqrt{r^2} \\ 10 &= r \end{aligned}$$

An equation for the circle is $x^2 + y^2 = 100$, and the radius of the circle is 10.

b) Consider a circle with its centre at the origin and with point B(−5, 9) on the circumference. Let r_1 be the radius of this circle. To find the length of the radius, substitute the coordinates of point B into the formula for the radius of a circle centred at the origin.



$$\begin{aligned} r_1 &= \sqrt{x^2 + y^2} \\ &= \sqrt{(-5)^2 + 9^2} \\ &= \sqrt{25 + 81} \\ &= \sqrt{106} \\ &\doteq 10.3 \end{aligned}$$

Since $r_1 > 10$, point B lies outside the circle defined by $x^2 + y^2 = 100$.

If $r_1 > r$, then $r_1^2 > r^2$. So, the inequality $x^2 + y^2 > r^2$ defines the region *outside* the circle with centre (0, 0) and radius r.

Reflect